Designing Overhanging Joists

by Harris Hyman, P.E.

This article is the second of two (the first ran in March '99), working through the structure of a Garrison with its characteristic second-story overhang. The second-story joist must carry not only the interior loads, but also the load from the roof and the exterior wall on the overhanging end (see Figure 1).

In the first article we calculated the loads on the second-story joists. There are four distinct loads:

- the interior dead load from the second level structure
- the interior live load from people and furniture in the upstairs space
- the dead load onto the outside end of the joist from the roof and exterior wall structure
- the live load from snow on the roof

But to build it, we need details: Are the joists to be 2x8s 24 inches on-center or 2x12s 8 inches on-center? Are they to be Spruce-Pine-Fir or Doug fir? Construction grade, #1, or select structural?

The usual way to work out the details is to guess at a suitable joist and then check it out under the loadings. If this

Loads on a Garrison Joist

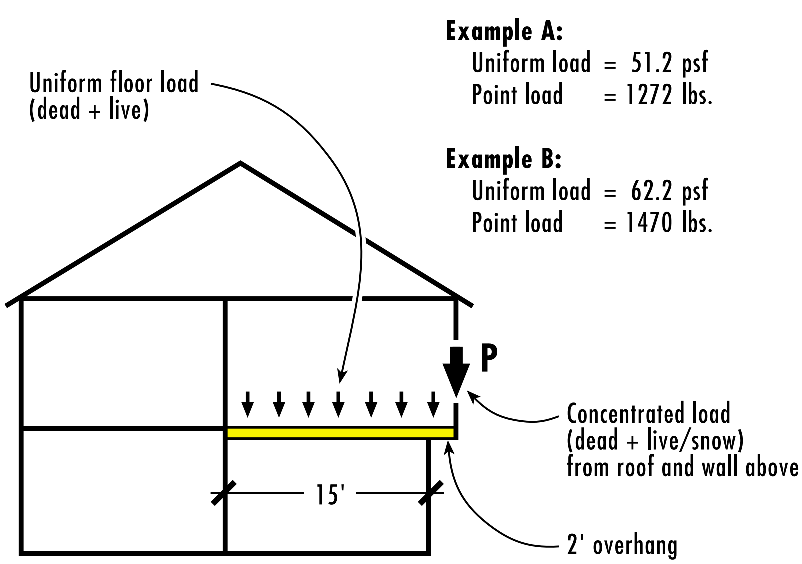


Figure 1. There are two types of loads on an overhanging joist like the ones in a Garrison Colonial, shown here. The weight of the materials in the roof and upper exterior wall, plus the snow load on the roof, accumulate as a point load on the end of the joist. Inside, the joist must also carry the dead load of the floor structure plus the occupant live load. See the March '99 *Practical Engineering* column for the derivation of the loads given as examples A and B.

test joist is overstressed or if it deflects too much, a heavier stick is needed. If the joist proves way too strong, perhaps a lighter one could be used, saving money and resources. The designer typically tries a few selections until the optimal one is found.

Loading Conditions

The design check should cover all possible loading conditions. Unlike a simple joist that has only one loading condition, the Garrison joist has three to consider (Figure 2).

First is the winter storm condition, with snow all over the roof and the interior fully occupied. This is the worst loading case, with the joists bearing the brunt of the full dead and live loads, both uniform and concentrated.

Second is the fair weather condition, with the joist supporting the interior live and dead loads and also the dead load of the roof and wall structure, but with no snow load on the roof. If we can size a joist to handle the first case, we should be able to safely ignore this condition.

The third load case is the empty house in winter. Inside, the joist carries no live load, only the dead load of the building materials. Again, the first loading case seems to be more severe and should control the design. But there's one thing we'll need to check here: Without the occupant load on the inside, the concentrated snow load on the outboard end exerts a lifting effect on the joist — a seesaw action. We should determine how much lifting force is exerted and what it takes to resist it at the other end of the joist.

Finding the Worst Case Loading Conditions

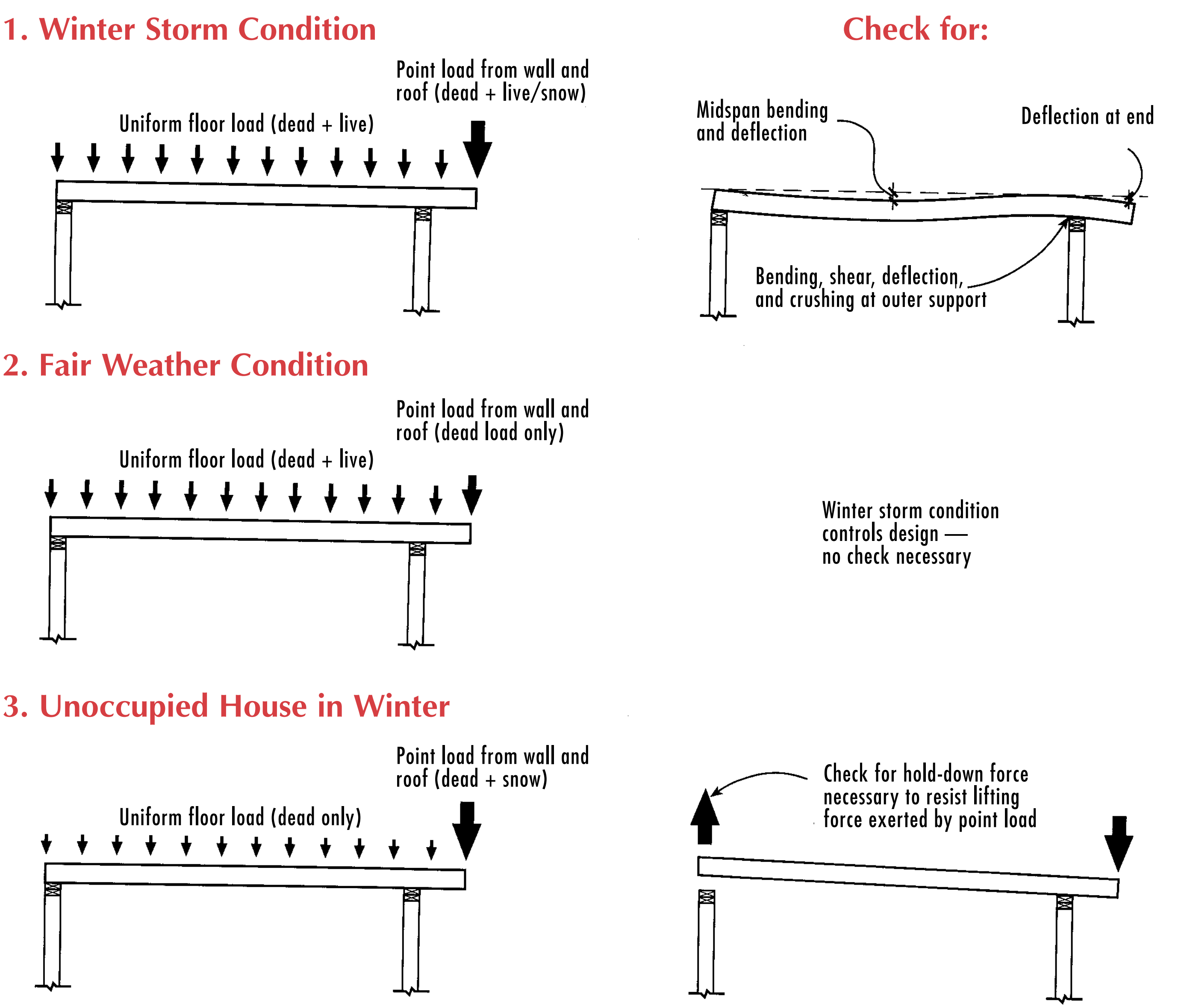


Figure 2. A careful beam design must be based on the worst case loading scenario — in this case, the winter storm condition, with all the loads piled on at once. Condition 3, the unoccupied house in winter, must also be considered for the lifting effect the snow load exerts on the joist.

Beam Diagrams

The basic tool for evaluating a beam is the set of beam diagrams. This is a collection of sketches of various beam loading and support situations, accompanied by formulas for calculating reaction, shear, bending moment, and deflection. Beam diagrams are reproduced in a variety of references; I'm currently using the *Wood Structural Design Data* (American Forest & Paper Assoc.; 202/463-2733).

The Garrison joist is in the category called a "Beam Overhanging One Support." (In the language of beam diagrams, it's not called a "cantilever," as most carpenters would refer to it.) The two loading conditions that concern us here are the "Uniformly Distributed Load" for the live and dead load inside the house, and the "Concentrated Load at End of Overhang" for the wall and roof loads coming down on the outboard end (Figure 3).

Unfortunately, there is no diagram in the reference books that combines uniform loading and concentrated loading. In some instances, we'll be adding uniform and point load effects together.

Positive & Negative Moments

Look at the bending moment diagrams in Figure 3. For the "Uniformly Distributed Load," the moment rises gradually and peaks somewhere near the middle of the span, then falls to a *negative* peak at the first-floor support wall. A positive bending moment — drawn on top of the line representing the beam — means that the top edge of the beam is in compression and the bottom edge is in tension, as is the case with most beams in a house. A load on the overhang, however, puts the top edge of the beam in tension.

The bending moment for the "Concentrated Load at End of Overhang" is always negative, coming to a peak at the exterior wall. In the center of the main span, the two moments tend to cancel; that is, the point load causes a negative moment, while the uniform load has a positive moment. Right over the exterior wall, however, the moments add, because both are negative.

We'll be conservative in the design calculations, considering only the uniform load moment for the midspan stress, without subtracting out the effect of the concentrated load. At the outside support — the other peak load point — we'll use the sum of the two moments over the wall.

Designing a Joist

The joist must pass three tests. It must be sufficiently strong in bending so that it will not break, sufficiently strong in shear so that it will not tear apart along the grain, and sufficiently stiff so that it will not deflect too far. We'll use the load calculations from the previous article and work through example A. To keep this article from getting too long, I'll give you the answers for example B and let you try your hand at the calculations to see if you get there.

Our first step is to select a joist: I'll try 2x10s first, 16 inches on-center. I'm in the West, so I'll use No. 1 Douglas fir.

Table A: Dimension Lumber Properties

Lumber Size	Section Modulus (in.^{3})	Moment of Inertia (in.^{4})
2x6	7.6	20.8
2x8	13.1	47.6
2x10	21.4	98.9
2x12	31.6	178.0

Bending Calcs

Typically, the first check is for bending. For this, we calculate the bending moment according to the diagram, then calculate the bending stress by another formula. We then compare the stress on the loaded beam to the *allowable* stress, which is a published design value.

First we'll calculate the midspan moment from the uniform floor load. Next we'll total up the moments at the outer support wall from both the uniform and point loads on the overhang. Our guess is that the moment at the support wall will be larger than the midspan moment, but we'd better check. We almost always design for the worst case.

Now pull out your March issue of *JLC*. From the previous article, the uniform floor load for example A is 51.2 psf. For joists 16 inches (1.33 ft.) on-center, this is 68 psf per foot of joist (51.2 x 1.33). We'll convert feet to inches from the start, because design stresses are given in pounds per square inch. Thus

$\boldsymbol{w}$ = 68 psf / 12 in./ft
= 5.7 lb. per inch of joist

P = 1,272 lb.

ℓ = 15 ft. × 12 in./ft. =180 in.

a = 2 ft. × 12 in./ft. = 24 in.

Midspan moment uniform load:

$$M_1 = \frac{w(\ell + a)^2\,(\ell - a)^2}{8\ell^2}$$

$$= \frac{5.7 \times (180 + 24)^2 \times (180 - 24)^2}{8 \times 180^2}$$

$$= 22{,}271 \text{ in. lb.}$$

Exterior support moment, uniform load:

$$M_2 = \frac{w\,a^2}{2}$$

$$= \frac{5.7 \times 24^2}{2} = 1{,}642 \text{ in. lb.}$$

Exterior support moment, concentrated load:

$$M_{max} = P \times a$$
$$= 1{,}272 \times 24 = 30{,}528 \text{ in. lb.}$$

Total moment at support wall:

$$M_{tot} = 1{,}642 + 30{,}528 = 32{,}170 \text{ in. lb.}$$

The moment at the exterior wall is larger, so it controls the design. In other words, if we size the joist to handle this moment, the lumber will also be strong enough for the midspan moment.

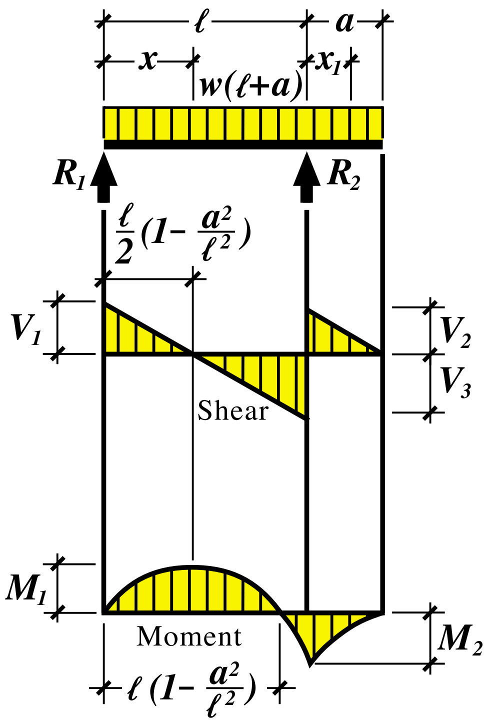
Bending stress. Now that we know the maximum bending moment, we can calculate the bending stress (f_b), using the section modulus (S) for a 2x10, which we can look up in a table (see Table A), or calculate with the formula

$$S = \frac{bd^2}{6}$$

where b is the width of the lumber and d is the depth. For now we look it

Diagrams & Formulas for Overhanging Beams

Beam Overhanging One Support — Uniformly Distributed Load



$R_1 = V_1$ . . . . $= \frac{w}{2\ell}(\ell^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . $= \frac{w}{2\ell}(\ell + a)^2$

V_2 . . . . $= wa$

V_3 . . . . $= \frac{w}{2\ell}(\ell^2 + a^2)$

V_x (between supports) . . . . $= R_1 - wx$

V_{x_1} (for overhang) . . . . . . $= w(a - x_1)$

$M_1 \left(\text{at } x = \frac{\ell}{2}\left[1 - \frac{a^2}{\ell^2}\right]\right)$ . . . $= \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$

M_2 (at R_2) . . . . $= \frac{wa^2}{2}$

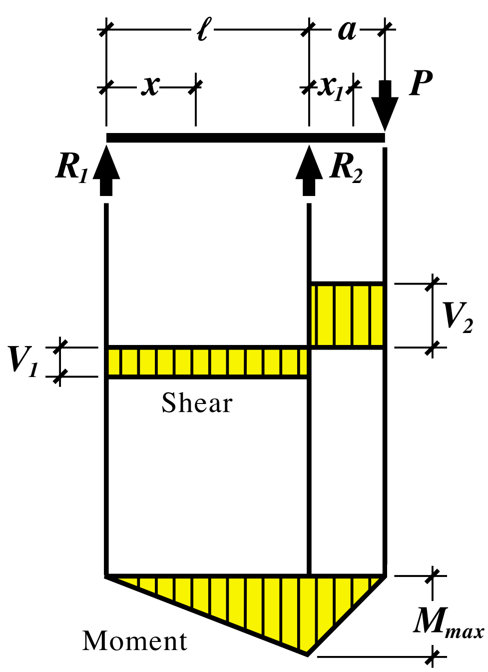
M_x (between supports) . . . . $= \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$

M_{x_1} (for overhang) . . . . . . $= \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) . . . . $= \frac{wx}{24EI\ell}(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)$

Δ_{x_1} (for overhang) . . . . . . $= \frac{wx_1}{24EI}(4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

Beam Overhanging One Support — Concentrated Load at End of Overhang



$R_1 = V_1$ . . . . $= \frac{Pa}{\ell}$

$R_2 = V_1 + V_2$ . . . . $= \frac{P}{\ell}(\ell + a)$

V_2 . . . . $= P$

M_{max} (at R_2) . . . . $= Pa$

M_x (between supports) . . . . $= \frac{Pax}{\ell}$

M_{x_1} (for overhang) . . . . $= P(a - x_1)$

$\Delta_{max} \left(\text{between supports at } x = \frac{\ell}{\sqrt{3}}\right) = \frac{Pa\ell^2}{9\sqrt{3}EI} = .06415\frac{Pa\ell^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) . . . $= \frac{Pa^2}{3EI}(\ell + a)$

Δ_x (between supports) . . . . $= \frac{Pax}{6EI\ell}(\ell^2 - x^2)$

Δ_{x_1} (for overhang) . . . . $= \frac{Px_1}{6EI}(2a\ell + 3ax_1 - x_1^2)$

Figure 3. Beam diagrams with accompanying formulas are used to calculate reaction and shear forces, bending moment, and expected deflection under load.

up; for a 2x10, it's 21.4 in.^{3}. The bending stress formula is:

$$f_b = \frac{M}{S}$$

Plugging in the numbers:

$$f_b = \frac{32{,}170}{21.4} = 1{,}503 \text{ psi}$$

This is the actual bending stress, but what is the *allowable* bending stress (F_b)? We look up #1 Doug fir in the *NDS* (the *National Design Specification for Wood Construction* — a must-have if you're going to do wood beam sizing) and find that the allowable stress value is only 1,000 psi (see Table B). This figure can be increased by the Repetitive Member factor, a load-sharing allowance for joists and rafters that are set no more than 24 inches on-center. The factor is 1.15, which increases the allowable F_b:

$F_b' = 1{,}000 \times 1.15 = 1{,}150$ psi.

This is still too small, since the actual bending stress in the beam is 1,503 psi. We have to make a choice: We can set the joists closer, so there are more of them sharing the load; we can use a stronger lumber; or we can use a deeper joist, which will increase the section modulus. Let's stay with #1 Doug fir and 16-inch centers, but try 2x12s, which have a section modulus of 31.6. With the deeper section, the actual bending stress is now:

$$f_b = \frac{32{,}170}{31.6} = 1{,}018 \text{ psi}$$

Since the allowable bending stress, F_b', is greater, at 1,150 psi, the 2x12s will do the job.

Shear Calculations

We're okay for bending, so we go on to shear stress (V). Referring to the diagrams, the maximum negative shear from both the uniform and point loads is at the outboard support. We'll calculate and add them together. First, for the uniform load, V_3 (see beam diagrams) is the worst case of the three shearing forces, and the one we'll need to calculate:

$$V_3 = \frac{w(\ell^2 + a^2)}{2\ell}$$

Table B: Allowable Design Values for Common Lumber Species

Species	Grade	F_b	F_v	E	$F_{c\perp}$
Douglas Fir-Larch	Select Structural	1500	95	1,900,000	625
	No. 1 & Better	1200	95	1,800,000	625
	No. 1	1000	95	1,700,000	625
	No. 2	900	95	1,600,000	625
Hem-Fir	Select Structural	1400	75	1,600,000	405
	No. 1 & Better	1100	75	1,500,000	405
	No. 1	975	75	1,500,000	405
	No. 2	850	75	1,300,000	405
Spruce-Pine-Fir	Select Structural	1250	70	1,500,000	425
	No. 1/No. 2	875	70	1,400,000	425
	No. 3	500	70	1,200,000	425
Southern Pine	Select Structural	1900	90	1,800,000	565
	No. 1	1250	90	1,700,000	565
	No. 2	975	90	1,600,000	565

Source: 1997 *NDS*

$$= \frac{5.7 \times (180^2 + 24^2)}{2 \times 180} = 522 \text{ lb.}$$

For the Concentrated Load,

$V_2 = P = 1{,}272$ lb.

Totaling,

$V_{tot} = 522 + 1{,}272 = 1{,}794$ lb.

Now we can calculate the actual shear stress (f_v) with the formula:

$f_v = 1.5$ V/A

where A is the area of the 2x12, or 16.9 sq. in. So:

$$f_v = \frac{1.5 \times 1{,}794}{16.9} = 159.2 \text{ psi}$$

Next, we look up the allowable shear stress (F_v), which for Doug fir is 95 psi. This is a conservative base value that applies even to lumber that has long splits in the end — as long as 1$\frac{1}{2}$ times the width of the board or longer (17 inches for a 2x12). For lumber with shorter splits or no splits at all, the *NDS* allows you to increase the allowable shear — by a factor of 2 for no splits, by 1.67 for splits half as long as the width of the board, and by 1.5 for splits $\frac{3}{4}$ the width.

We'll assume the careful carpenter culls out the good sticks, and apply a factor of 1.67:

$F_v' = 95$ psi $\times 1.67 = 159$ psi

Close enough!

Deflection

The 2x12s are okay for shear, so now we calculate the deflection, *d*. Since the interior load causes a sag and the concentrated load lifts the joist a little, let's be conservative and consider *only* the distributed load, checking the point of maximum deflection between the supports. We'll need two new design values, *E*, the modulus of elasticity, and *I*, the moment of inertia. From the tables for Doug fir, $E = 1{,}700{,}000$ psi. The moment of inertia, like the section modulus, is based on the geometry of the dimension lumber. For a 2x12, $I = 178$ in.^{4} (the value can be calculated with the formula $I = bd^3/12$, but it's easier to look up.)

There's no formula for maximum deflection, but we know the point of maximum deflection is near the middle of the span. So we'll assume "x" in the deflection formula equals 90 in., half the back span.

$$d = \frac{wx\,(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)}{24EI\ell}$$

$$= \frac{5.7 \times 90\,(180^4 - (2 \times 180^2 \times 90^2) + (180 \times 90^3) - (2 \times 24^2 \times 180^2) + (2 \times 24^2 \times 90^2))}{24 \times 1{,}700{,}000 \times 178 \times 180}$$

$$= 0.25 \text{ in.}$$

Code calls for deflection of floors to be less than 1/360 the length of the span ($\ell/d \geq 360$). Plugging in our predicted deflection:

ℓ / d = 180/0.25 = 720 > 360 (okay!)

Final Checks

So we check out on the first load case, the winter storm. We can safely ignore the second load case, the fair weather condition. We have to consider the third condition, the empty house in winter, where the snow load wants to lift up the inboard end of the joist. The required hold-down force is equal to R_1, the support reaction at that end.

$$R_1 = \frac{Pa}{\ell} = \frac{1272 \times 24}{180} = 170 \text{ lb.}$$

This hold-down force can easily be provided by installing a simple metal hold-down between the top plate and the joist. Toenails must not be used here, since they'd be loaded in withdrawal.

Crushing. The final check is for crushing where the joist crosses the outside wall. Is the load so large that it will cause the wood fibers to compress and the floor to settle slightly? This is rarely a problem, but because we've got an unusually large outboard point load, we'd better make sure. The crushing force is equal to the sum of the reactions (R_2) from the uniform and point loads. Referring to the beam diagrams, for the uniform load:

$$R_2 = V_2 + V_3 = \frac{w(\ell + a)^2}{(2\ell)}$$

$$= \frac{5.7 \times (180 + 24)^2}{(2 \times 180)} = 659 \text{ lb.}$$

For the point load:

$$R_2 = V_1 + V_2 = \frac{P(\ell + a)}{\ell} =$$

$$= 1{,}272\,\frac{(180 + 24)}{180} = 1{,}442 \text{ lb.}$$

The total crushing load under the worst case winter storm condition is:

659 + 1,442 = 2,101 lb.

This crushing load doesn't come down on one point; it is distributed over an area equal to the thickness of the joist (1.5 in.) times the width of at least a 2x4 top plate (3.5 in.), or 5.25 sq. in. Dividing, the distributed crushing load is:

2,101 ÷ 5.25 psi = 400 psi

We compare this actual load to another allowable design value, compression perpendicular to grain, or $F_{C\perp}$. For Doug fir, $F_{C\perp}$ is 625 psi, so we're okay. Even if the top plate is a softer wood like S-P-F, the allowable value is 425 psi, so we're still okay.

Try It On Your Own

So that's it — a long tedious calculation. I took a couple of shortcuts here (to prevent a magazine article from becoming a thesis), purposely ignoring the Load Duration Factor, which allows an increase in allowable stress under snow loads. Also, deflection of the overhang was ignored; it's quite small in this case, though as the overhang grows longer, this deflection becomes more of an issue.

I actually used to work it out this way, and most of you — using the code, the beam formulas, and the *NDS* — can grind through the work. (I now use some fairly expensive software to do the job.)

Here are the results of example B from the March article, should you want to try your hand at the calculations:

The maximum moment, at the support wall, is 37,267 in. lb., which gives a bending stress of 1179 psi. This is 3% overstress (versus the allowable stress of 1150). The shear stress calculates out at 187 psi — higher than in example A, and requiring perfectly clean lumber with no splits. The deflection at midspan under uniform floor load is 0.3 in., giving a deflection ratio of ℓ/600 — plenty stiff!

In a case like this, where the stresses are very close to the maximum allowed, you have to make a choice: (1) Say "close enough" and just accept the overstress; (2) assume that the careful carpenter will cull the best joists out of the woodpile, meaning *we actually have #1 and Better*, which can handle a higher stress; or (3) move the joists to 12 inches on-center. My personal pick would be (2), but this is a subjective judgement. Approach (3) meets the letter of the law and the cost isn't too great, so it might be more comfortable all around — especially if a code official is looking over your shoulder.

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